Determination of Poisson's Ration and the Modulus of Elasticity by measuring with P- and S-wave transducers.

Required steps before measurements can be performed:

1. Put a small amount of shear wave coupling gel on the transducers.
2. Firmly press the transducers on either side of the 25 µs calibration rod (Part No 710 10 028). Make sure that the coupling gel is properly distributed and that no air is trapped between the transducer and the calibration rod.
3. Connect the transducers to Pundit Lab.
4. Select the 250 kHz transducer from the list of supported transducers (see Pundit Lab manual chapter 3 for more details).
5. Zero the instrument as described in the Pundit Lab manual chapter 2.1.

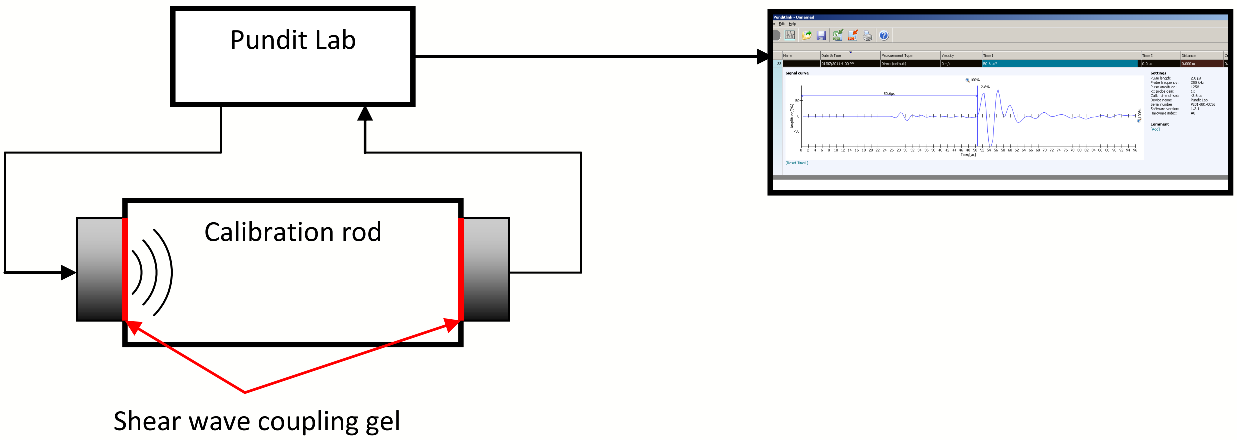


Figure 1: Performing measurements with the 250 kHz shear wave transducers.

Performing measurements (see Figure 1):

When measurements with the 250 kHz shear wave transducers are performed, it is crucial to use the special shear wave coupling paste, otherwise shear waves cannot be properly transmitted into the object under test. The shear wave coupling paste is a non-toxic, water soluble organic substance of very high viscosity.

Furthermore, we highly recommend using Pundit Link's waveform display in order to **manually** locate the onset of the shear wave echo. Since the latter is always preceded by a relatively weak longitudinal echo (see Figure 2), the transit time determined by Pundit Lab, would correspond to the longitudinal instead of the shear wave.

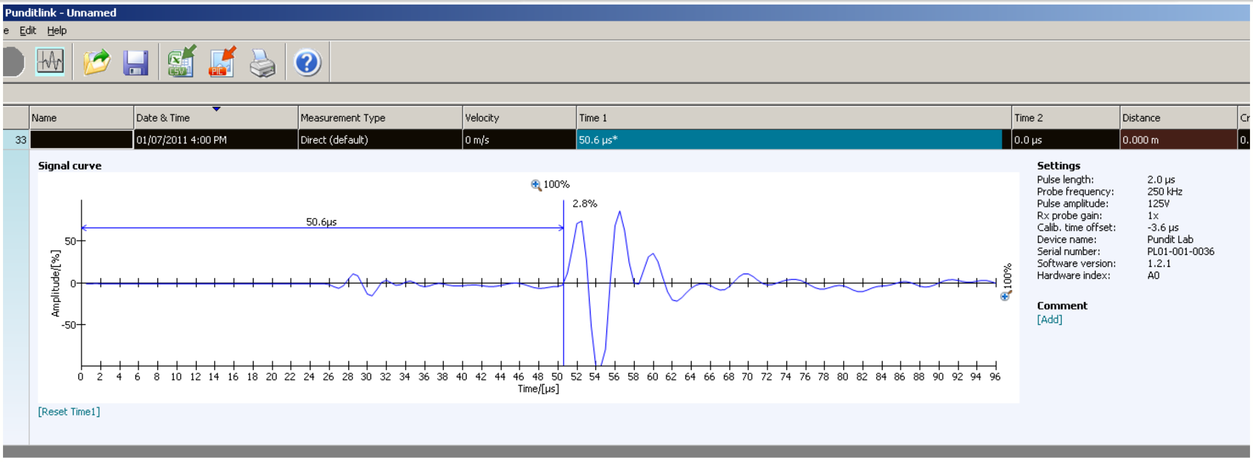


Figure 2: Typical echo signal obtained, with an experimental setup according to figure 1. The first echo arrives at approximately 25.4 µs and corresponds to the weak longitudinal wave generated by the shear wave transducer. After 50.6 µs the much stronger shear wave echo appears in the signal.

Using P and S wave measurements to determine Poisson's Ratio and Modulus of Elasticity:

This table taken from Wikepedia shows how elastic properties of materials may be determined, provided that two are known.

v • d • e **Elastic moduli for homogeneous isotropic materials** [hide]

Bulk modulus (K) • Young's modulus (E) • Lamé's first parameter (λ) • Shear modulus (G) • Poisson's ratio (ν) • P-wave modulus (M)

Conversion formulas [hide]

Homogeneous isotropic linear elastic materials have their elastic properties uniquely determined by any two moduli among these, thus given any two, any other of the elastic moduli can be calculated according to these formulas.

	(λ, G)	(E, G)	(K, λ)	(K, G)	(λ, ν)	(G, ν)	(E, ν)	(K, ν)	(K, E)	(M, G)
$K=$	$\lambda+\frac{2G}{3}$	$\frac{EG}{3(3G-E)}$			$\frac{\lambda(1+\nu)}{3\nu}$	$\frac{2G(1+\nu)}{3(1-2\nu)}$	$\frac{E}{3(1-2\nu)}$			$M-\frac{4G}{3}$
$E=$	$\frac{G(3\lambda+2G)}{\lambda+G}$		$\frac{9K(K-\lambda)}{3K-\lambda}$	$\frac{9KG}{3K+G}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	$2G(1+\nu)$		$3K(1-2\nu)$		$\frac{G(3M-4G)}{M-G}$
$\lambda=$		$\frac{G(E-2G)}{3G-E}$		$K-\frac{2G}{3}$		$\frac{2G\nu}{1-2\nu}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{3K\nu}{1+\nu}$	$\frac{3K(3K-E)}{9K-E}$	$M-2G$
$G=$			$\frac{3(K-\lambda)}{2}$		$\frac{\lambda(1-2\nu)}{2\nu}$		$\frac{E}{2(1+\nu)}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	$\frac{3KE}{9K-E}$	
$\nu=$	$\frac{\lambda}{2(\lambda+G)}$	$\frac{E}{2G}-1$	$\frac{\lambda}{3K-\lambda}$	$\frac{3K-2G}{2(3K+G)}$					$\frac{3K-E}{6K}$	$\frac{M-2G}{2M-2G}$
$M=$	$\lambda+2G$	$\frac{G(4G-E)}{3G-E}$	$3K-2\lambda$	$K+\frac{4G}{3}$	$\frac{\lambda(1-\nu)}{\nu}$	$\frac{2G(1-\nu)}{1-2\nu}$	$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$	$\frac{3K(1-\nu)}{1+\nu}$	$\frac{3K(3K+E)}{9K-E}$	

By measuring a P-wave transmission time and an S-wave transmission time with Pundit Lab, we are able to determine the P-wave modulus (M) and the Shear modulus (G).

P-wave modulus (M):

$$\boldsymbol{M = \rho V_p^2}$$

Where ρ is the density of the material and V_p is the pulse velocity of the P-wave.

Shear-modulus (G):

$$\boldsymbol{G = \rho V_S^2}$$

Where ρ is the density of the material and V_s is the pulse velocity of the S-wave.

Using the equations above we can determine Poisson's Ratio (v):

$$\boldsymbol{v} = \frac{M - 2G}{2M - 2G} = \frac{\rho V_P^2 - 2\rho V_S^2}{2\rho V_P^2 - 2\rho V_S^2} = \frac{V_P^2 - 2V_S^2}{2V_P^2 - 2V_S^2} = \frac{\boldsymbol{V_P^2 - 2V_S^2}}{\boldsymbol{2(V_P^2 - V_S^2)}}$$

So Poisson's ratio can be determined simply by measuring the P-wave velocity and the S-wave velocity and it is not even necessary to know the density of the material.

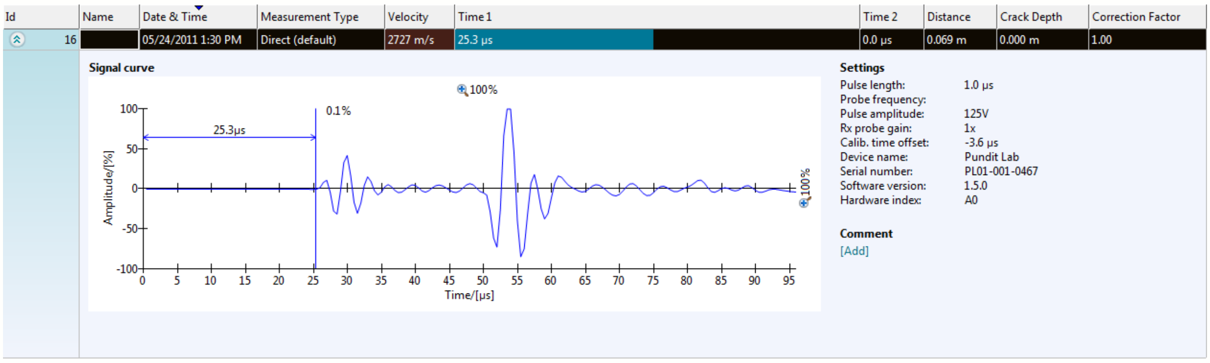
Once Poisson's ratio is known, the elastic modulus can be calculated from the equation:

$$\boldsymbol{E = 2G(1 + v)}\ .$$

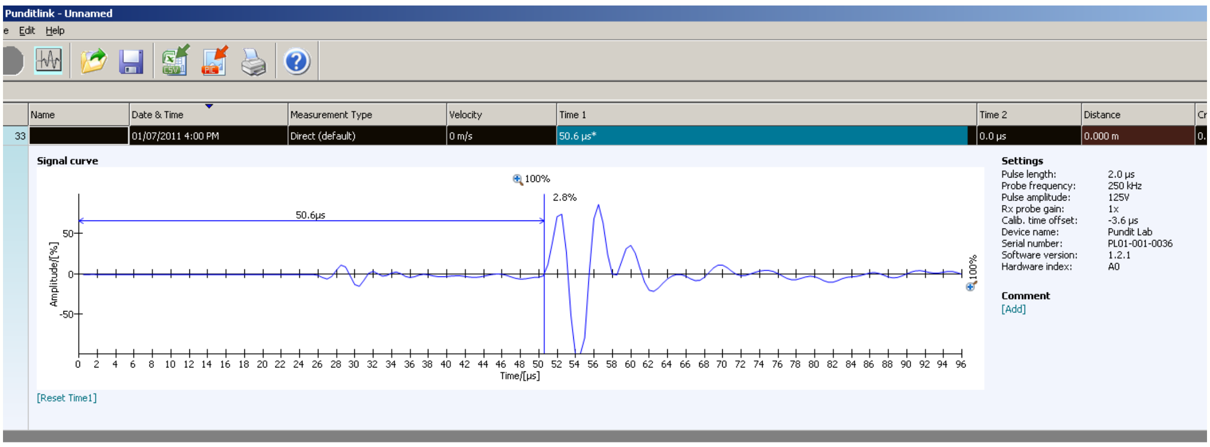
For this it is necessary to know the density of the material.

Practical Example made on the calibration rod:

The calibration rod is made of a material called PMMA:
E-modulus of PMMA is typically 2700–3200 MPa.
Density of PMMA is 1.18 g/cm^3.
Poisson's ratio for PMMA is typically 0.35 – 0.4.
The measured length of the calibration rod is 69 mm.



Measured P-wave transmission time is 25.3µs.



Measured S-wave transmission time is 50.6 µs.

Calculation of Poisson's ratio:

$$\nu = \frac{V_P^2 - 2V_S^2}{2(V_P^2 - V_S^2)}$$

V_P = 0.069/0.0000253 = 2727 m/s

V_S = 0.069/0.0000504 = 1369 m/s

Putting these figures into the equation above gives:

ν = 0.33

Note 1: For V_P = **2**V_S, Poisson's ratio **ν** is always **0.33**

Note 2: Typical Poisson's ratio for concrete is in the order of **0.06** to **0.27**

Calculation of the shear modulus G:

$$G = \rho V_S^2$$

G **=** 1180 x 1369^2 = 2.21 GPa

Calculation of the elastic modulus E:

$$E = 2G(1 + \nu)\ .$$

E = 2x2.21(1+0.33) = 5.88 GPa. = 5880 MPa